

Structure of a Fermion

							spin	iso-spin	flavour- chirality	particle	chirality		$2N$	$-3T_3$	$6Y$
		F					+	+	+	+	-1	ν_R	0	0	0
	σ_1	F					-	+	+	+	-1				
		F	σ_1				+	-	+	+	+1	e_L	-6	-3	-3
	σ_1	F	σ_1				-	-	+	+	+1				
		F		c_1			+	+	+	-	-1	$\bar{d}_R$	+2	0	+2
	σ_1	F		c_1			-	+	+	-	-1				
		F	σ_1	c_1			+	-	+	-	+1	$\bar{u}_L$	-4	-3	-1
	σ_1	F	σ_1	c_1			-	-	+	-	+1				
		F		c_2			+	+	+	+	-1	u_R	+4	0	+4
	σ_1	F		c_2			-	+	+	+	-1				
		F	σ_1	c_2			+	-	+	+	+1	d_L	-2	-3	+1
	σ_1	F	σ_1	c_2			-	-	+	+	+1				
		F		c_3			+	+	+	-	-1	$\bar{e}_R$	+6	0	+6
	σ_1	F		c_3			-	+	+	-	-1				
		F	σ_1	c_3			+	-	+	-	+1	$\bar{\nu}_L$	0	-3	+3
	σ_1	F	σ_1	c_3			-	-	+	-	+1				
τ_1		F					+	+	-	+	+1	ν_L	0	+3	-3
τ_1	σ_1	F					-	+	-	+	+1				
τ_1		F	σ_1				+	-	-	+	-1	e_R	-6	0	-6
τ_1	σ_1	F	σ_1				-	-	-	+	-1				
τ_1		F		c_1			+	+	-	-	+1	$\bar{d}_L$	+2	+3	-1
τ_1	σ_1	F		c_1			-	+	-	-	+1				
τ_1		F	σ_1	c_1			+	-	-	-	-1	$\bar{u}_R$	-4	0	-4
τ_1	σ_1	F	σ_1	c_1			-	-	-	-	-1				
τ_1		F		c_2			+	+	-	+	+1	u_L	+4	+3	+1
τ_1	σ_1	F		c_2			-	+	-	+	+1				
τ_1		F	σ_1	c_2			+	-	-	+	-1	d_R	-2	0	-2
τ_1	σ_1	F	σ_1	c_2			-	-	-	+	-1				
τ_1		F		c_3			+	+	-	-	+1	$\bar{e}_L$	+6	+3	+3
τ_1	σ_1	F		c_3			-	+	-	-	+1				
τ_1		F	σ_1	c_3			+	-	-	-	-1	$\bar{\nu}_R$	0	0	0
τ_1	σ_1	F	σ_1	c_3			-	-	-	-	-1				
							$\rho_3 1$	$1 \rho_3$	$L_7 1$	$1 L_7$	$L_1 \rho_3$				

					spin	iso-spin	flavour-chirality	particle	chirality		$2N$	$-3T_3$	$6Y$
τ_1		F			+	+	-	+	+1	ν_L	0	+3	-3
τ_1	σ_1	F			-	+	-	+	+1				
		F	σ_1	c_2	+	-	+	+	+1	d_L	-2	-3	+1
	σ_1	F	σ_1	c_2	-	-	+	+	+1				
τ_1		F		c_2	+	+	-	+	+1	u_L	+4	+3	+1
τ_1	σ_1	F		c_2	-	+	-	+	+1				
		F	σ_1		+	-	+	+	+1	e_L	-6	-3	-3
	σ_1	F	σ_1		-	-	+	+	+1				
		F			+	+	+	+	-1	ν_R	0	0	0
	σ_1	F			-	+	+	+	-1				
τ_1		F	σ_1	c_2	+	-	-	+	-1	d_R	-2	0	-2
τ_1	σ_1	F	σ_1	c_2	-	-	-	+	-1				
		F		c_2	+	+	+	+	-1	u_R	+4	0	+4
	σ_1	F		c_2	-	+	+	+	-1				
τ_1		F	σ_1		+	-	-	+	-1	e_R	-6	0	-6
τ_1	σ_1	F	σ_1		-	-	-	+	-1				
		F	σ_1	c_3	+	-	+	-	+1	$\bar{\nu}_L$	0	-3	+3
	σ_1	F	σ_1	c_3	-	-	+	-	+1				
τ_1		F		c_1	+	+	-	-	+1	$\bar{d}_L$	+2	+3	-1
τ_1	σ_1	F		c_1	-	+	-	-	+1				
		F	σ_1	c_1	+	-	+	-	+1	$\bar{u}_L$	-4	-3	-1
	σ_1	F	σ_1	c_1	-	-	+	-	+1				
τ_1		F		c_3	+	+	-	-	+1	$\bar{e}_L$	+6	+3	+3
τ_1	σ_1	F		c_3	-	+	-	-	+1				
τ_1		F	σ_1	c_3	+	-	-	-	-1	$\bar{\nu}_R$	0	0	0
τ_1	σ_1	F	σ_1	c_3	-	-	-	-	-1				
		F		c_1	+	+	+	-	-1	$\bar{d}_R$	+2	0	+2
	σ_1	F		c_1	-	+	+	-	-1				
τ_1		F	σ_1	c_1	+	-	-	-	-1	$\bar{u}_R$	-4	0	-4
τ_1	σ_1	F	σ_1	c_1	-	-	-	-	-1				
		F		c_3	+	+	+	-	-1	$\bar{e}_R$	+6	0	+6
	σ_1	F		c_3	-	+	+	-	-1				
					$\rho_3 1$	$1 \rho_3$	$L_7 1$	$1 L_7$	$L_1 \rho_3$				

64 states in total (32 excluding spin). L and R refer to $SU(2)_L$ active vs inactive.

$$\begin{aligned}
\sigma_1 &= L_7 \rho_1 & \sigma_2 &= L_7 \rho_2 & \sigma_1 \sigma_2 &= -\rho_3 \\
\tau_1 &= L_{1,2,3} & \tau_2 &= L_{23,31,12} & \tau_1 \tau_2 &= -L_7
\end{aligned}$$

$$\begin{aligned}
X &= L_{1,23} + L_{2,31} + L_{3,12} \\
c_0 &= \{1, L_7\} \times \{1, L_{1,23}, L_{2,31}, L_{3,12}\} \\
c_1 &= c_0 \times \{L_{23}, L_{31}, L_{12}\} \\
c_2 &= c_1 \times L_{23,31,12} \\
c_3 &= c_0 \times L_{23,31,12} \\
C &= \frac{1}{4}(1 + L_7 X) = \frac{1}{2}(1 - L_7 R_7) = -L_7 \frac{1}{2}(L_7 + R_7) \\
S &= \frac{1}{2}(1 - L_7 \rho_3) = -L_7 \frac{1}{2}(L_7 + \rho_3) \\
F &= SC = -\frac{1}{2}(L_7 + \rho_3) \frac{1}{2}(L_7 + R_7) \\
f &= L_7 |\rho_3 \quad \text{flavour-active } \pm \\
\chi &= \frac{1}{2}(1 + f) \quad \text{flavour-active projector}
\end{aligned}$$

$$\begin{aligned}
\Lambda_4 &= \frac{1}{2}(L_{31,3} + L_{12,2}) \\
\Lambda_1 &= \frac{1}{2}(L_{12,1} + L_{23,3}) \\
\Lambda_6 &= \frac{1}{2}(L_{23,2} + L_{31,1}) \\
-\Lambda_5 &= \frac{1}{2}(L_{2,3} + L_{31,12}) \\
\Lambda_2 &= \frac{1}{2}(L_{3,1} + L_{12,23}) \\
\Lambda_7 &= \frac{1}{2}(L_{1,2} + L_{23,31}) \\
\Lambda_3 &= \frac{1}{2}(L_{1,23} - L_{3,12}) \\
-\Lambda_8 &= \frac{1}{2\sqrt{3}}(L_{1,23} + L_{3,12} - 2L_{2,31}) = \frac{1}{2}(\frac{1}{\sqrt{3}}X - \sqrt{3}L_{2,31}) \\
N &= \frac{1}{2}(3|\rho_3 - 1|X) \\
3Q &= N \\
T_1 &= \chi(\tau_1 |\bar{\rho}_1) \\
T_2 &= \chi(\tau_1 |\bar{\rho}_2) \\
T_3 &= \chi(1|\bar{\rho}_3) = T_1 T_2 = \frac{1}{2}(L_7 |1 - 1|\rho_3) \\
6Y &= 2N + 3T_3
\end{aligned}$$

$i\Lambda_k$ are the canonical SU(3) generators.

Transformations

$\rho_3 1$	space-axial spin,
$\sigma_1 1$	spin up/down
$1 \rho_3$	iso-axial spin
$1 \sigma_1$	iso-spin up/down
$L_7 1$	flavour-axial spin
$\tau_1 1$	flavour-spin up/down, γ_0
$\tau_1 \tau_1$	flip of L_7 , complex conjugation, exchange particle/anti-particle
$1 \{L_{23}, L_{31}, L_{12}\}$	R-G-B-spin
$1 L_7$	RGB/energy spin ($\pm$ for particle/anti-particle)
$1 c_i$	color alignment, color parity: particle/anti-particle
$L_7 L_7\rho_3$	chiral spin, γ_{0123}
$\mathbb{H} 1$	Lorentz Spin(3) symmetry, $\gamma_{23}, \gamma_{31}, \gamma_{21}$
$L_7 \mathbb{H} 1$	Lorentz boost symmetry, $\gamma_{10}, \gamma_{20}, \gamma_{30}$
$1 \Lambda_k$	$SU(3)_C$ color symmetry
T_i	$SU(2)_L$ flavour symmetry
N	$U(1)_{EM}$ axial symmetry
Y	$U(1)_Y$ axial symmetry
$\{\tau_1, L_7\} 1$	Higgs
$-L_7 \rho_3$	chirality (inactive (+) vs active (-))
$\frac{1}{2}(1 + L_7 \rho_3)$	chirality projector
$1 1, \tau_1 \sigma_1$	flavour-inactive
$\tau_1 1, 1 \sigma_1$	flavour-active

SU(5)

$$\begin{aligned}
SU(3)_C &= \begin{pmatrix} 0 & & \\ & -i\Lambda_k & \\ & & \end{pmatrix} & SU(2)_L &= \begin{pmatrix} -iT_i & \\ & 0 \end{pmatrix} \\
6Y &= \begin{pmatrix} 3 & & & & \\ & 3 & & & \\ & & -2 & & \\ & & & -2 & \\ & & & & -2 \end{pmatrix} & -3T_3 &= \begin{pmatrix} -3 & & & & \\ & 3 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \\
6Q = 6Y - 3T_3 &= \begin{pmatrix} 0 & & & & \\ & 6 & & & \\ & & -2 & & \\ & & & -2 & \\ & & & & -2 \end{pmatrix} & f &= \begin{pmatrix} d \\ u \\ r \\ g \\ b \end{pmatrix} \cong \begin{pmatrix} \varphi_0 \\ \varphi_+ \\ H_r \\ H_g \\ H_b \end{pmatrix}
\end{aligned}$$

Bits	Particle	6Q	-3T ₃	6Y
	$\bar{n}_L$	0	0	0
	$\bar{n}_R$	0	3	3
	$\bar{e}_R$	6	-3	3
	$\bar{e}_L$	6	0	6
	d_R	-2	0	-2
	d_L	-2	3	1
	u_L	4	-3	1
	u_R	4	0	4
	d_R	-2	0	-2
	d_L	-2	3	1
	u_L	4	-3	1
	u_R	4	0	4
	$\bar{u}_L$	-4	0	-4
	$\bar{u}_R$	-4	3	-1
	$\bar{d}_R$	2	-3	-1
	$\bar{d}_L$	2	0	2
	d_R	-2	0	-2
	d_L	-2	3	1
	u_L	4	-3	1
	u_R	4	0	4
	$\bar{u}_L$	-4	0	-4
	$\bar{u}_R$	-4	3	-1
	$\bar{d}_R$	2	-3	-1
	$\bar{d}_L$	2	0	2
	$\bar{u}_L$	-4	0	-4
	$\bar{u}_R$	-4	3	-1
	$\bar{d}_R$	2	-3	-1
	$\bar{d}_L$	2	0	2
	e_R	-6	0	-6
	e_L	-6	3	-3
	n_L	0	-3	-3
	n_R	0	0	0