From *A Treatise on Electricity and Magnetism.*

$$\mathfrak{B} = V.\nabla\mathfrak{A} = \nabla\mathfrak{A} \tag{1}$$
$$\mathfrak{E} = V.\mathfrak{G}\mathfrak{B} - \dot{\mathfrak{A}} - \nabla\Psi \tag{2}$$
$$\mathfrak{F} = V.\mathfrak{C}\mathfrak{B} + \mathfrak{e}\mathfrak{E} - \mathfrak{m}\nabla\Omega \tag{3}$$
$$\mathfrak{B} = \mathfrak{H} + 4\pi\mathfrak{I} \tag{4}$$
$$4\pi\mathfrak{C} = V.\nabla\mathfrak{H} \tag{5}$$
$$\mathfrak{K} = C\mathfrak{E} \tag{6}$$
$$\mathfrak{D} = \frac{1}{4\pi}K\mathfrak{E} \tag{7}$$
$$\mathfrak{C} = \mathfrak{K} + \dot{\mathfrak{D}} \tag{8}$$
$$\mathfrak{B} = \mu\mathfrak{H} \tag{9}$$
$$\mathfrak{e} = S.\nabla\mathfrak{D} \tag{10}$$
$$\mathfrak{m} = S.\nabla\mathfrak{I} \tag{11}$$
$$\mathfrak{H} = -\nabla\Omega \tag{12}$$

- $\mathfrak{A}$: electromagnetic momentum at a point
- $\mathfrak{B}$: magnetic induction
- $\mathfrak{C}$: total electric current
- $\mathfrak{D}$: electric displacement
- $\mathfrak{E}$: electromotive intensity
- $\mathfrak{F}$: mechanical force
- $\mathfrak{G}$: velocity of a point
- $\mathfrak{H}$: magnetic force
- $\mathfrak{I}$: intensity of magnetization
- $\mathfrak{K}$: current of conduction
- ψ: electic potential
- Ω: magnetic potential
- $\mathfrak{e}$: electric density
- $\mathfrak{m}$: density of magnetic 'matter'
- C: conductivity for electric currents
- K: dielectric inductive capacity
- μ: magnetic inductive capacity

$$\nabla^2\mathfrak{A} = 4\pi\mu\mathfrak{C}$$
$$S.\nabla\mathfrak{A} = 0$$

Gauss' law for magnetism (1):

$$\begin{aligned}\boldsymbol{B} &= \nabla \times \boldsymbol{A} \\ \nabla \cdot \boldsymbol{B} &= \nabla \cdot (\nabla \times \boldsymbol{A}) = 0\end{aligned}$$

Gauss' law (10), (7):

$$\begin{aligned}\nabla \cdot \boldsymbol{D} &= \rho \\ \nabla \cdot (\frac{1}{4\pi} K \boldsymbol{E}) &= \rho \\ \nabla \cdot \boldsymbol{E} &= \frac{\rho}{\epsilon_0}\end{aligned}$$

Ampère's law (5), (9), (8), (7):

$$\begin{aligned}\nabla \times \boldsymbol{H} &= 4\pi \boldsymbol{C} \\ \nabla \times \boldsymbol{B} &= 4\pi\mu \boldsymbol{C} \\ &= 4\pi\mu(\boldsymbol{J} + \dot{\boldsymbol{D}}) \\ &= \mu_0 \left(\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)\end{aligned}$$

Faraday's law (2), (1):

$$\begin{aligned}\boldsymbol{E} &= -\dot{\boldsymbol{A}} - \nabla\phi \\ &= -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\phi \\ \nabla \times \boldsymbol{E} &= -\nabla \times \frac{\partial \boldsymbol{A}}{\partial t} - \nabla \times (\nabla\phi) \\ &= -\frac{\partial(\nabla \times \boldsymbol{A})}{\partial t} \\ &= -\frac{\partial \boldsymbol{B}}{\partial t}\end{aligned}$$

Some relabellings:

$$\begin{aligned}\rho &= \mathfrak{e} \\ \epsilon_0 &= \frac{K}{4\pi} \\ \mu_0 &= 4\pi\mu \\ \boldsymbol{J} &= \mathfrak{K} \\ \phi &= \Psi\end{aligned}$$

If we fully embrace quaternions, the four equations can be reduced to two. Note $\nabla \boldsymbol{A} = -\nabla \cdot \boldsymbol{A} + \nabla \times \boldsymbol{A}$, thus:

$$\begin{aligned}\nabla \boldsymbol{E} &= -\frac{\rho}{\epsilon_0} - \frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \boldsymbol{B} &= \mu_0 \left(\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)\end{aligned}$$

Or with natural units ($\epsilon_0 = \mu_0 = 1$) and slight change of notation:

$$\begin{aligned}\nabla \boldsymbol{E} + \partial_t \boldsymbol{B} &= -\rho \\ \nabla \boldsymbol{B} - \partial_t \boldsymbol{E} &= \boldsymbol{J}\end{aligned}$$